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Nonperturbative Type I–I' String Theory

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Abstract

We propose a nonperturbative framework for the $O(32)$ type I open and closed string theory. The short distance degrees of freedom are bosonic and fermionic hermitian matrices belonging respectively to the adjoint and fundamental representations of the special unitary group $SU(N)$. We identify a closed matrix algebra at finite N which corresponds to the Lorentz, gauge, and supersymmetry algebras of the large N continuum limit. The planar reduction of our matrix theory coincides with the low energy spacetime effective action of the $d=10$ type I $O(32)$ unoriented open and closed string theory. We show that matrix T -duality transformations can yield a nonperturbative framework for the T -dual type I' closed string theory with 32 D8branes. We show further that under a strong-weak coupling duality transformation the large N reduced action coincides with the low energy spacetime effective action of the $d=10$ heterotic string, an equivalence at leading order in the inverse string tension and with either gauge group $Spin(32)/Z_2$ or $E_8 \times E_8$. Our matrix formalism has the potential of providing a nonperturbative framework encapsulating all of the weak coupling limits of M theory.

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The consistent perturbative quantization of Yang-Mills theories with chiral matter fields coupled to gravity provided by either the type I or heterotic string theories— at weak coupling, and in a framework which is anomaly-free, and both ultraviolet and infrared finite, is a remarkable achievement [1, 2]. Nonetheless, in the absence of a nonperturbative framework, there remains open both the question of fundamental principle [3], and the danger that the properties of the nonperturbative ground state of string theory turn out to be drastically different at even a qualitative level [4, 5]. In addition, many outstanding problems of particle physics, such as vacuum selection, spacetime supersymmetry breaking, and the generation of hierarchies in the Standard Model, are widely assumed to only find resolution in nonperturbative physics at very high energy scales, possibly even Planckian. Thus, it is a matter of crucial importance to discover the correct framework for nonperturbative string theory.

Candidates for such a nonperturbative framework have been proposed based on the large N quantum mechanics of D0branes [5, 7]: “particles” carrying both electric, and solitonic, charge, that have been proposed as the fundamental constituents of both the solitons and strings of M theory in [7]. An alternative starting point is the planar reduction of ten-dimensional super Yang-Mills theory giving the zero-dimensional matrix action of [8]. A third, and closely related, proposal is matrix string theory, based on the large N limit of two-dimensional $SU(N)$ gauge theory [9]. All three proposals are Wilsonian cut-off theories [19]. The implicit assumption has been made that off-diagonal $(1/N)$ quantum corrections to the planar action representing massive terms will, upon integrating out, provide missing relevant interactions in the classical action required in order to match correctly with the known infrared limit [4, 3]. While the detailed matching between supergravity and matrix computations is difficult, with considerable room for ambiguity, the success of either proposal in meeting this crucial consistency check remains in doubt [11].

More seriously, a clear-cut demonstration of spacetime Lorentz invariance in the large N limit is lacking in both the BFSS model and in matrix string theory as a consequence of an inherently light-cone formulation which obscures many points of physics [7, 9]. The IKKT model has different shortcomings. An explicit demonstration of dynamically selected eigenvalue configurations characterizing the large N ground state has given evidence for a nonperturbative mechanism for vacuum selection with fewer than ten noncompact spacetime dimensions [10]. While incompletely understood at the moment, this is intriguing. On the other hand, the arguments for making contact with the $\mathcal{N}=2$ spacetime supersymmetries of the ten-dimensional IIB string, and with its Dstring ground states, are shaky. A clear separation of Ramond-Ramond sector solitons from the Neveu-Schwarz sector solitons of the supersymmetric gauge theory has not been made. Perhaps the most significant aspect of the IKKT proposal, we will argue, is the insight it offers into the dilemma of how to preserve manifest spacetime diffeomorphism invariance in a fully quantum theory of gravity.

The IKKT action is the dimensional reduction of $d=10$ $\mathcal{N}=1$ $SU(N)$ supersymmetric Yang-Mills theory to a point [17, 8], or “event”, in spacetime. Not surprisingly, it coincides with the classical worldvolume action of N coincident Dinstantons [5, 3]. While this is hardly the motivation for the IKKT proposal, we comment that it provides a simple understanding for the absence of a mass scale in the IKKT matrix action [8]: the Dpbrane tension scales as $\tau_p = O(g^{-1}\alpha'^{-(p+1)/2})$, dimensionless when $p=-1$ [5]. It also provides an understanding of extensions to the IKKT action since such terms should have a natural interpretation in terms of the full quantum non-abelian Born-Enfield action [3]. We will, however, take a somewhat different path towards a candidate nonperturbative

matrix theory by first incorporating a crucial feature absent in both the BFSS and IKKT matrix actions. Namely, we replace the $N \times N$ fermionic, Grassmann-valued, matrices with Grassmann-valued matrices transforming in the N and \bar{N} fundamental representations of the $SU(N)$ group. This allows us to incorporate chirality into a matrix theory with sixteen supercharges, giving rise to sixteen component spinor fields in the large N limit that can simultaneously satisfy both the Majorana and Weyl conditions of the $d=10$ $\mathcal{N}=1$ SYM-supergravity theory [21, 1, 2, 22].

Motivated by this insight, we consider a $SU(N)$ invariant hermitian matrix action with sixteen supercharges. The bosonic matrices live in the adjoint representation of the special unitary group, while the sixteen component chiral fermionic matrices transform in the N -dimensional fundamental representations of $SU(N)$. The components of a bosonic hermitian matrix belong to the field of real, or complex, numbers while those of a fermionic matrix belong to the field of anticommuting Grassmann numbers. The continuum large N limit of our theory is defined as follows: we take $\epsilon = M_{11}^{-1} \rightarrow 0$, where M_{11} is the eleven-dimensional Planck mass, such that the length scale ϵN is held fixed, providing a short distance cutoff in our theory [19]. The continuum limit is therefore an expansion about a ground state with diagonal entries for all matrices, such that in the large N limit objects in the adjoint and fundamental representations evolve, respectively, into the continuum bose and fermi fields of a continuum action. This continuum action will be matched with the spacetime low energy effective action of the heterotic-type I string theory at leading order in the α' expansion. The short distance cutoff of the large N theory can be expressed in terms of the ten-dimensional closed string tension by the relation: $\epsilon N = N e^{\frac{2}{3}\Phi_0} \alpha'^{1/2}$, where Φ_0 is the vev of the continuum dilaton field in the large N ground state.

An important distinction from [7, 9] is incorporation of a key feature of the original motivation behind the Eguchi-Kawai reduction of a large N gauge theory [17]: the necessity for the explicit appearance of a spacetime continuum will be abandoned. Invoking translational invariance in the noncompact directions and the gauge symmetries will enable reduction of the number of dynamical degrees of freedom in the nonperturbative supersymmetric gauge-gravity theory to those on a single site of the spacetime lattice. Notice that spacetime diffeomorphism invariance follows automatically in any nonperturbative matrix formulation of a quantum theory of gravity based on *zero-dimensional* matrices. It is only in the specification of the ground state, namely, the large N limit about which we expand when computing the $O(1/N)$ quantum corrections arising in the matrix path integral, that diffeomorphism invariance is spontaneously broken. Upon taking the large N continuum limit, we must refer all quantum fields to a fixed background spacetime metric. Thus, the perturbative string theories, and their low energy effective field theory limits, describe the long-distance fluctuations of matter-energy in a spacetime continuum, but with respect to a fixed background metric, and in a ground state with spontaneously broken diffeomorphism invariance. Notice that we preserve the spirit of Einstein's classical theory of general relativity: spacetime geometry is replaced by an equivalent matter-energy distribution together with the Einstein equations [23]. But given both the matter fields and a specific solution to the classical equations enables recovery, at least in principle, of a corresponding spacetime metric. This is also true for the nonperturbative matrix quantum theory of gravity. The target spacetime geometry is abandoned in favour of matrix degrees of freedom. However, in any large N ground state of the theory, diffeomorphism invariance will be *spontaneously* broken: a necessary feature of the long-distance effective description of the gauge-gravity interactions provided by the string theories [1, 2, 3].

In this paper, we arrive at a proposed matrix action for nonperturbative string theory by the process of *inference*, from a knowledge of both the target-space, and strong-weak coupling, dualities [4, 5], and the specific form of the low energy spacetime effective action of the type I open and closed unoriented string theory [21, 2, 22, 3]. Our methodology obviates the difficulties encountered in making a precise match between S-matrix amplitudes of the matrix model and of supergravity theory [11]. It simultaneously incorporates in the large N continuum limit both the matter (Yang-Mills) and supergravity sectors of the anomaly-free ten-dimensional type I string. It also gives a clear demonstration of Lorentz invariance and of manifest supersymmetric and non-abelian gauge covariance in the large N continuum theory. But, best of all, we find that the classical matrix action thereby obtained has an elegant and simple structure that can be motivated largely from first principles, and by symmetry considerations alone. Perhaps this is not surprising given the aesthetic use of symmetry principles— both kinematic and dynamical, in the original formulation of perturbative string theory [1, 2, 3, 12], now appearing in the large N limit of a hermitian matrix action.

Our starting point is an action with $N \times N$ bosonic hermitian matrices transforming in the adjoint representation of the special unitary group, coupled to sixteen component fermionic hermitian matrices in the N -dimensional fundamental representations of $SU(N)$. In what follows, we work in natural units setting $\hbar = c = 1$. The $SU(N)$ matrix variables may carry, in addition, both Lorentz and nonabelian group indices. Matching the short-distance cutoff ϵN with the string scale, we infer the form of matrix action required in order that the large N limit yields a continuum effective field theory action exhibiting $\mathcal{N}=1$ spacetime supersymmetry, local Lorentz invariance, and Yang-Mills gauge symmetry, and with the anomaly-free massless field content of the $O(32)$ type I string [19, 3]. For convenience, we denote the finite-dimensional Yang-Mills group as the generic group \mathbf{G} , of rank r_G , and dimension d_G , although the reader can assume for the purposes of this paper that \mathbf{G} is the rank 16 group $SO(32)$ (or $E_8 \times E_8$, see [12]). As is well-known, upto suitable identifications under heterotic-type I strong-weak coupling duality [4, 6, 3], the large N continuum action also corresponds to the low energy spacetime effective action of the $d=10$ $Spin(32)/Z_2$ heterotic string [2, 3].

The inner product in the Hilbert space for any hermitian matrix variables is defined by the direct multiplication of matrices, while taking care to preserve $SU(N)$ invariance. Since the matrix variables can, in addition, carry Lorentz indices, a matrix ordering prescription compatible with the continuum Lorentz transformations of the fields obtained in the large N limit is necessary. Specifically, we must distinguish left- and right- matrix multiplication within any given bilinear of matrices. We fix the ordering ambiguity by recognizing the $SL(2, C)$ decomposition of each Lorentz tensor: an object in the $(n, 0)$, or $(0, \bar{n})$, representation is a left-multiplier, while the $(0, n)$, or $(\bar{n}, 0)$ representation multiplies from the right. Thus, anti-spinors precede spinors in any matrix bilinear, and contravariant tensors precede covariant tensors. By “spinor” and “anti-spinor” here, we mean fermionic, Grassmann-valued, matrices: objects transforming in the fundamental and anti-fundamental, the N and \bar{N} representations of $SU(N)$, evolving, respectively, into spinor and anti-spinor continuum fields in the large N limit. Likewise, for the generic covariant and contravariant matrix Lorentz “tensors”. It is helpful to write down explicit expressions for the simplest cases. Consider the inner product for a matrix Lorentz “tensors” defined so as to exhibit manifest invariance under finite N matrix Lorentz transformations. We use tangent space indices

whenever possible, avoiding explicit appearance of the matrix “vierbein”, E_a^μ , both in the action, and in the inner product. Tangent space indices are raised and lowered by the η^{ab} and ϵ^{ab} symbols. We note the following simple identities for generic covariant and contravariant matrix “tensors”:

$$\begin{aligned} \eta^{ab} &= E^{a\mu} E_\mu^b, & \eta_{ab} &= E_a^\mu E_{b\mu}, & A^\mu A_\mu &= A^a E_a^\mu E_{b\mu} A^b = A^a A_a \\ A^\mu &= A^a E_a^\mu, & A_\mu &= E_{b\mu} A^b, & \bar{\psi}_\mu &= \bar{\psi}_a E_\mu^a, & \psi_\mu &= E_\mu^a \psi_a \\ A^{\mu\nu} A_{\mu\nu} &= A^{ca} E_a^\nu E_c^\mu E_{d\mu} E_{b\nu} A^{db} = A^{ca} E_a^\nu \eta_{cd} E_{b\nu} A^{db} = A^{ca} \eta_{ab} A_c^b = A^{ca} A_{ca} \quad , \end{aligned} \quad (1)$$

and likewise for the higher rank cases. The ordering of matrices in the inner product is defined unambiguously. Notice the natural correspondence between the trace of a given matrix bilinear of $SU(N)$ matrices with the Lorentz invariant inner product for a continuum field obtained in the large N limit [18, 8]. We have:

$$\begin{aligned} (\bar{\Psi}, \Psi') &\equiv \text{tr } \bar{\Psi} \Psi' &\rightarrow \int d^{10}x \bar{\Psi}(x) \Psi'(x) \\ (A_{[1]}, A'_{[1]}) &\equiv \text{tr } A^\mu A'_\mu = \text{tr } A^a A'_a &\rightarrow \int d^{10}x A^a(x) A'_a(x) \\ (A_{[2]}, A'_{[2]}) &\equiv \text{tr } A^{\mu\nu} A'_{\mu\nu} = \text{tr } A^{ab} A'_{ab} &\rightarrow \int d^{10}x A^{ab}(x) A'_{ab}(x) \quad , \end{aligned} \quad (2)$$

and likewise for higher rank cases. The trace denotes a sum over the diagonal elements of the composite matrix operator. The continuum fields $\Psi(x)$, $A_a(x)$, and $A_{ab}(x)$, transform, respectively, as a d=10 tangent space spinor, vector, and two-form tensor field. Notice that each matrix “tensor” appearing in a matrix bilinear is rescaled by a power of the matrix “vierbein”, $E^{1/4}$, consistent with the appearance of diffeomorphism invariance in the large N continuum limit.

To illustrate the basic methodology by which we will proceed, recall the well-known case of the $N \times N$ hermitian one-matrix model [13]:

$$\mathcal{S}_B = \text{Tr } \Phi \Omega^a \Omega_a \Phi, \quad \Delta \equiv \Omega^a \Omega_a, \quad \Omega_a \Phi_n = \lambda_{na}^{1/2} \Phi_n, \quad n = 1, \dots, N \quad . \quad (3)$$

We are assuming here that $\Delta = \Omega^a \Omega_a$ is a self-adjoint linear operator Δ , with orthonormalized eigenfunctions, $\{\Phi_n\}$, and eigenvalues, $\{\lambda_n\}$. This will indeed be true for all of the “kinetic” terms in our matrix action. In the large N limit, we will make the usual extension to a Hilbert space with continuous eigenfunctions $\{\phi(\lambda; \mathbf{x})\}$. Note that the dependence on the underlying target space, \mathbf{x} , is only implicit; we work in a generalized momentum basis, or with suitable phase space variables. The transition from finite N to infinite N could be ill-defined. Given the nature of the eigenvalue spectrum of Δ in the large N limit, appropriate low and high momentum regulators may be needed in order to make the integral well-defined. Alternatively, one can imagine modifying the large N limit, as was done in the well-known case of the double-scaling limit for string theories with superconformal matter of central charge $c \leq 3/2$ [15]. In the cases $c < 3/2$, where there is no one-dimensional embedding superspace, the eigenvalue density variable obtained in the large N limit can be identified with the super-Liouville modes expected from two-dimensional superconformal supergravity [14, 15]. This equivalence followed from the remarkable isomorphism observed between the graphical expansion for triangulated random surfaces and the dual planar graph expansion of the one-matrix model with generic polynomial potential [13, 15]. Notice that no symmetry other than $SU(N)$ invariance, employed here in order to diagonalize the matrix and thereby decouple the angle variables [13], has been invoked in analytically treating the one-matrix path integral. What happens if the matrix action is characterized by additional symmetry?

In the case of gauge theories, recall that we can invoke gauge invariance in performing the path integration over orbits of the gauge group, extracting the group volume a la Faddeev-Popov, and accounting separately for any potential infrared divergences arising from the integration over the gauge slice. In the case of the hermitian one-matrix model [13], this was straightforwardly implemented both at finite and at infinite N , since both the action and the inner product localizes on the diagonal elements of the $N \times N$ matrix. In the presence of interactions coupling two, or more, species of matrices, an analytic treatment of zero-dimensional matrix path integrals is less obvious, although the conceptual underpinnings are similar [13]. More generally, in order to perform the path integration over matrix variables we must specify an inner product invariant under the extended symmetry group \mathcal{G} of matrix transformations that leave the classical action invariant: let \mathcal{G} define the finite N remnant of the Lorentz, gauge, and supersymmetry invariances of the continuum large N action. Such finite N symmetry transformations have nontrivial overlap with the group of $SU(N)$ rotations. Since they constitute a “gauge invariance” of our matrix theory, \mathcal{G} invariance should be manifest both in the classical action, and in the inner product defined on the Hilbert space of matrix “tensors”. We must be careful in defining the group invariant measure in the matrix path integral, where by “group” we mean here the semi-direct product group $\mathcal{G} \times SU(N)$ acting on the hermitian matrix “tensors”. Since every term in the continuum large N action is required to be singlet under both matrix Lorentz, \mathcal{L} , and matrix Yang-Mills, \mathbf{G} , transformations, we will identify matrix composites invariant under the finite N matrix manifestation of these symmetries. We comment that this procedure is reminiscent of equivariant localization [16], an observation of likely note considering the matrix theory can in many respects be considered topological, in the sense that it has no intrinsic length scale. Having briefly clarified the motivation for identifying matrix algebra analogs of the continuum Lorentz, gauge, and supersymmetry invariances of the effective field theories describing the large N ground state, we proceed to an analysis of the classical matrix action.

Our expression for the classical matrix action will be manifestly invariant under $\mathcal{L} \times \mathbf{G}$. With guidance from the low energy continuum type I string effective action [21, 22, 3], it is not difficult to infer its form:

$$\begin{aligned} \mathcal{S} = & \frac{1}{\kappa^2} \left(\bar{\psi}_a \Gamma^{abc} D_b \psi_c - 4 \bar{\lambda} \Gamma^{ab} D_a \psi_b - 4 \bar{\lambda} \Gamma^a D_a \lambda \right) + g^2 e^\Phi \bar{\chi}^i \Gamma^a D_a \chi^i \\ & + g^2 e^\Phi F^{ab} F_{ab} + \frac{1}{\kappa^2} \left(\mathcal{R} - 4 \Omega^a \Phi \Omega_a \Phi + 3 e^{2\Phi} H^{abc} H_{abc} \right) \\ & + \mathcal{S}_{2\text{-fermi}} + \mathcal{S}_{4\text{-fermi}} \quad . \end{aligned} \tag{4}$$

Spinor and $SU(N)$ indices have been suppressed in this expression, and the notation is as follows. In comparing with the low energy string effective action [22], note that we have absorbed the overall factor of $E^{1/2} e^{-2\Phi}$ in the definition of the $SU(N)$ trace, also omitting an overall minus sign in the action. This will be of importance when we give a detailed prescription for taking the continuum limit with some, or all, coordinates noncompact. It is conventional in the literature to write the continuum low energy spacetime effective action with an additional overall factor of $\frac{1}{2}$ [3], also dropped in our matrix action. Notice that explicit dilaton dependence in the measure is restricted to the kinetic terms for the Yang-Mills, and two-form, matrix “tensors”. This is a result of our rescaling of the $SU(N)$ trace.

In the expression above, $\chi^{i\alpha}$, $\bar{\chi}^{i\alpha}$, denote Grassmann-valued fermionic matrices in the N , \bar{N} , representations of $SU(N)$. The indices, $i=1, \dots, r_G$, simultaneously labels a fundamental repre-

sentation of the Yang-Mills group \mathbf{G} , while $\alpha=1, \dots, 16$, labels sixteen distinct Grassmann-valued fermionic matrices, evolving in the large N continuum limit into the sixteen components of a Majorana-Weyl spinor field. Likewise, ψ_μ^α , denotes Grassmann-valued fermionic matrices evolving in the large N limit into the components of a Lorentz spinor-vector field in ten dimensions. Finally, we have the matrix representatives of the dilatino field, also in Grassmann-valued $SU(N)$ fundamental representations, λ^α . In the continuum limit, χ^i , ψ_μ , and λ , yield, respectively, the gaugino, gravitino, and dilatino fields of the d=10 $\mathcal{N}=1$ SYM supergravity action. The $SU(N)$ matrices F_{ab} , H_{abc} , \mathcal{R} , and Φ are, respectively, finite N matrix representatives of the Yang-Mills tensor, the shifted antisymmetric three-form field strength corresponding to the two-form potential $C_{[2]}$, plus Chern-Simons term for the Yang-Mills potential, $A_{[1]}$, the Ricci curvature, and the dilaton scalar continuum fields.

The matrix operator $D_a \psi_b$ is required to evolve into a rank two covariant tensor in the continuum large N limit. In general, $\Gamma^a D_a$ may be expressed as an expansion in the complete set of independent Lorentz structures in the type II theories with sixteen component spinors:

$$(\Gamma^a D_a)_{\alpha\beta} \equiv \Omega_a (\Gamma^a)_{\alpha\beta} + \Omega_{ab} (\Gamma^a \Gamma^b)_{\alpha\beta} + \Omega_{abc} (\Gamma^a \Gamma^{bc})_{\alpha\beta} + \Omega_{abcd} (\Gamma^a \Gamma^{bcd})_{\alpha\beta} + \Omega_{abcde} (\Gamma^a \Gamma^{bcde})_{\alpha\beta} + \text{duals} \quad . \quad (5)$$

In the large N continuum limit, the matrix operator $\Gamma^a D_a$ is required to evolve into the appropriate bosonic, or fermionic, super-covariant derivative, with appropriate couplings, in general, to both spin connection and nonabelian vector potential. In principle, it could couple as well to the two, or magnetic dual six-form, gauge potentials of the nonperturbative type I string theory [5, 3]. Note that the dual couplings are not, however, necessitated by closure of the matrix Lorentz algebra. Most generally, including only the full spectrum of even potentials in the IIB theory, we could write:

$$\begin{aligned} \Gamma^a D_a \rightarrow & \left((\mathbf{1} + C_{(0)}(x)) \partial_a + A_a^j(x) \tau^j \right) \Gamma^a + (A_{ab}(x) + C_{ab}(x)) \Gamma^a \Gamma^b + \omega_{abc}(x) \Gamma^a \Gamma^{bc} \\ & + C_{abcd}(x) \Gamma^a \Gamma^{bcd} + \omega_{abcde}(x) \Gamma^a \Gamma^{bcde} \quad . \end{aligned} \quad (6)$$

The one-form, $A_a \equiv A_a^j \tau^j$, is the familiar Yang-Mills gauge potential. The τ^j are hermitian generators of the Yang-Mills gauge group \mathbf{G} , the f_{ijk} are its structure constants, $[\tau^i, \tau^j] = i f^{ijk} \tau^k$, and $g=e^\Phi$ is the dimensionless open string coupling. We note that this prescription for incorporating nonabelian gauge symmetry is similar in spirit to the suggestion in [8] that an $SU(K)$ Yang-Mills symmetry arises whenever eigenvalue configurations decompose naturally into block-diagonal clusters of K eigenvalues. Finally, $\omega_{abc}[e(x)]$ is the ordinary spin-connection including the dilaton dependent piece, expressed in terms of the vierbein in the first-order formalism for the low energy type I string effective action [21, 3].

The tensors, C_{ab} , and C_{abcdef} , represent gauge potentials coupling to, respectively, D1branes, or D5branes, when present in the IB large N ground state [5, 3]. The Φ dependence in their kinetic terms in the effective action will distinguish them from antisymmetric gauge potentials with the same number of Lorentz indices [3]. The two-form matrix “tensor”, $C_{[2]}$, although not required, is consistent with both the matrix supersymmetry and matrix Lorentz algebra as will be shown below. Finally, recall that the fluctuating parts of the continuum fields, A_{ab} , C_{abcd} , and $C_{(0)}$, are absent in the ten-dimensional type I unoriented string. We should emphasize that none of the antisymmetric tensor potentials are *necessary* for closure of the Lorentz algebra. Instead, they represent additional vacuum charges consistent with closure of the Lorentz algebra. These “Ramond-Ramond” charges

[5]—originally named for the fermionic vacuum in the sector of the perturbative string theory with periodic, or Ramond, boundary conditions on both left- and right-handed worldsheet fermions [3], become necessary only upon application of the weak-strong coupling duality principle to the string vacuum state [4, 5, 3].

Notice that the five-index antisymmetric matrix “tensor” is new, unanticipated from field theory considerations, although clearly permitted by symmetry considerations alone. The necessity of including such a term in the classical matrix action comes from closure of the matrix Lorentz algebra when acting on sixteen-component Majorana-Weyl spinors: a five-index matrix commutator, $[\Omega_{abc}, \mathbf{L}_{de}]$, appears naturally upon Lorentz transformation of the spin-connection. Thus, closure of the matrix Lorentz algebra for a generic Majorana-Weyl spinor, Ψ , *requires inclusion of a five-index antisymmetric matrix*, ω_{abcde} , in addition to the usual spin-connection. The supercovariant derivative operator may be defined as follows:

$$\Gamma^a D_a \Psi = \Gamma^a \left(\Omega_a + \frac{1}{2} \Omega_{abc} \Gamma^{bc} + \frac{1}{4} \Omega_{abcde} \Gamma^{bcde} \right) \Psi \quad . \quad (7)$$

It is easy to verify that in the presence of a matrix two-form “tensor” potential, C_{ab} , the nontrivial commutator with \mathbf{L}_{de} implies a necessity for both non-vanishing $C_{[0]}$, and $C_{[4]}$, matrix “tensors”. In the continuum large N limit, the matrix “field strengths” assume the familiar form of the corresponding fields:

$$\begin{aligned} R_{\mu\nu}^{ab}[\omega_{ab}^\mu] &\rightarrow \partial_\mu \omega_\nu^{ab}(x) - \partial_\nu \omega_\mu^{ab}(x) + \omega_\nu^{ac}(x) \omega_{\mu c}^b(x) - \omega_\mu^{ac}(x) \omega_{\nu c}^b(x) \\ F_{\mu\nu}^i[A_1] &\rightarrow \partial_\mu A_\nu^i(x) - \partial_\nu A_\mu^i(x) + f^{ijk} A_\mu^j(x) A_\nu^k(x) \\ H_{\mu\nu\rho}[C_{\mu\nu}, A_\mu] &\rightarrow \partial_{[\mu} C_{\nu\rho]} - \text{tr} (A_{[\mu} \partial_\nu A_{\rho]}) - \frac{2}{3} A_{[\mu} A_\nu A_{\rho]} \quad . \end{aligned} \quad (8)$$

$C_{[2]}$ is the two-form gauge potential of the type I string. The antisymmetric two-form matrix potential, A_{ab} , which ordinarily couples in the large N limit to fundamental, heterotic, or type II, oriented closed strings, is restricted to take constant values alone in ground states of the unoriented type I string. Likewise, acting on scalar, vector, and two-form, potentials, Ω_a is required to evolve into the familiar nonabelian gauge covariant derivative in the large N continuum limit:

$$\Omega_a \rightarrow \partial_a + A_a^j \tau^j \quad . \quad (9)$$

It is easy to verify closure of the matrix Lorentz algebra in the absence of p -form gauge potentials, $p \geq 2$, with this definition for Ω_a . Closure corresponds, in this case, to the full ten-dimensional Poincare group of spacetime symmetries characterizing the type I ground state with ten-dimensional $\mathcal{N}=1$ supergravity and $O(32)$ Yang-Mills fields in the large N continuum limit.

Returning to a complete description of the classical matrix action, we must include crucial two-fermion and four-fermion terms required by supersymmetry [21, 22]. With guidance from the continuum low energy type I string effective action [22], and keeping in mind the overall minus sign and factor of $\frac{1}{2}$ described above, we infer that the 2-fermi terms take the form:

$$\begin{aligned} \mathcal{S}_{2\text{-fermi}} &= 2 \bar{\psi}_a \Gamma^a \psi_b (\Omega^b \Phi) - 4 \bar{\psi}_a \Gamma^b \Gamma^a \lambda (\Omega_b \Phi) \\ &\quad - \frac{1}{2^3} H^{def} \left[\bar{\psi}_a \Gamma^{[a} \Gamma_{def} \Gamma^{b]} \psi_b + 4 \bar{\psi}_a \Gamma_{def}^a \lambda - 4 \bar{\lambda} \Gamma_{def} \lambda + g^2 e^\Phi \bar{\chi} \Gamma_{def} \chi \right] \\ &\quad + \frac{1}{4} g^2 e^\Phi \bar{\chi}^i \Gamma^d \Gamma^{ab} (\psi_d + \frac{1}{3} \Gamma_d \lambda) F_{ab}^i \quad . \end{aligned} \quad (10)$$

Likewise, the 4-fermi terms in the matrix classical action take the form:

$$\begin{aligned} \mathcal{S}_{4\text{-fermi}} &= \frac{1}{3 \cdot 2^6} \bar{\psi}^f \Gamma^{abc} \psi_f \left(\bar{\psi}_d \Gamma^d \Gamma_{abc} \Gamma^e \psi_e + 2 \bar{\psi}^d \Gamma_{abc} \psi_d - 4 \bar{\lambda} \Gamma_{abc} \lambda - 4 \bar{\lambda} \Gamma_{abc} \Gamma^d \psi_d \right) \\ &\quad + \frac{1}{3 \cdot 2^6} g^2 e^\Phi (\bar{\chi} \Gamma^{abc} \chi) \left(\bar{\psi}_d (4 \Gamma_{abc} \Gamma^d + 3 \Gamma^d \Gamma_{abc}) \lambda - 2 \bar{\lambda} \Gamma_{abc} \lambda - 3 \cdot 2^3 H_{abc} \right). \end{aligned} \quad (11)$$

As before, it can be verified that matrix Lorentz invariance holds for both the 2-fermi and 4-fermi terms in the classical matrix action. The expression for \mathcal{S} can be simplified and written in a more compact form by introducing $SU(N)$ vectors, Ψ , $\bar{\Psi}$, and the matrix operator, \mathcal{D} . The $(1+10+d_G)N$ -component $SU(N)$ vectors transform as, respectively, 16-component right- and left-handed Majorana-Weyl “spinors” under the matrix Lorentz group, and can be denoted as follows:

$$\bar{\Psi} \equiv (\bar{\lambda}, \bar{\psi}_a, \bar{\chi}^i), \quad \Psi \equiv (\lambda, \psi_b, \chi^j) \quad . \quad (12)$$

Likewise, we assemble the independent Lorentz structures in the kinetic and two-fermi terms of \mathcal{S} inside a matrix array of size, $(11+d_G)N \times (11+d_G)N$. Likewise, the four-fermi terms can be expressed in compact form by introducing matrices, \mathcal{U} , \mathcal{V} , of size $(11+d_G)N \times (11+d_G)N$, by referring to Eq. (11). Finally, we can define the matrix scalar “Laplacian”, $\Delta \equiv \Omega^a \Omega_a$. The matrix classical action can now be written in remarkably compact form:

$$\mathcal{S} = \frac{1}{2} \bar{\Psi} \mathcal{D} \Psi + \frac{1}{4} (\bar{\Psi} \mathcal{U} \Psi) (\bar{\Psi} \mathcal{V} \Psi) + \frac{1}{2} g^2 e^\Phi F^{ab} F_{ab} + \frac{1}{2} \frac{1}{\kappa^2} (\mathcal{R} - \frac{1}{2} \Phi \Delta \Phi + 3 e^{2\Phi} H^{abc} H_{abc}) \quad . \quad (13)$$

Under the matrix Lorentz group, Ψ and $(\Psi)^{*t}$ transform, respectively, in the $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ representations.

The precise form of the matrix action given above has as large N ground state the unoriented type I open and closed string theory in noncompact flat ten-dimensional spacetime [21, 3]. As explained earlier, in order for the large N ground state of the matrix theory to correspond to a string vacuum carrying additional charges due to the presence of Dbranes as part of the target spacetime geometry, we must include the appropriate kinetic and Chern-Simons terms for the matrix p -form potentials of the type I theory. We allow for the possibility of constant $A_{[2]}$. And, in addition to $C_{[2]}$, incorporated in the shifted three-form field strength, we must include kinetic terms for all $C_{a_1 \dots a_p}$, with p even, and $0 \leq p \leq 10$ [5, 3]. This is as required by the application of weak-strong coupling duality transformations on a given large N ground state. Notice that in the matrix T_9 -dual type I' ground state, the corresponding T_9 -dual matrix $p \pm 1$ -form potentials will appear. Thus, the matrix action for the T_9 -dual type I' nonperturbative string is easily inferred from a knowledge of the T_9 -duality transformations of the corresponding field in the large N limit effective field theory ground state [3].

Finally, let us verify closure of the group of matrix transformations that are the finite N manifestation of the Lorentz symmetries characterizing the large N continuum limit. We introduce an infinitesimal hermitian matrix, L_{ab} , antisymmetric under the interchange of tangent space indices a, b . Keeping terms upto linear in L_{ab} , it is easy to verify that the matrix action \mathcal{S} is invariant under matrix Lorentz transformations which we define as follows:

$$\begin{aligned} \delta \chi^i &= \Gamma^{ab} L_{ab} \chi^i, & \delta \bar{\chi}^i &= -\bar{\chi}^i \Gamma^{ab} L_{ab} \\ \delta \psi_a &= \Gamma^{bd} L_{bd} \psi_a + L_a^c \psi_c, & \delta \bar{\psi}_c &= -\bar{\psi}_c \Gamma^{ab} L_{ab} - \bar{\psi}_a L_c^a \\ \delta \lambda &= \Gamma^{bd} L_{bd} \lambda, & \delta \bar{\lambda} &= -\bar{\lambda} \Gamma^{ab} L_{ab} \end{aligned}$$

$$\begin{aligned}
\delta(\Gamma^a D_a) &= [\Gamma^{ab} L_{ab}, \Gamma^c D_c] \\
\delta(\Gamma^{ab} D_b) &= [\Gamma^{de} L_{de}, \Gamma^{ab} D_b] + L_d^a \Gamma^{db} D_b \\
\delta(\Gamma^{abc} D_b) &= [\Gamma^{de} L_{de}, \Gamma^{abc} D_b] - [\Gamma^{abd} D_b, L_d^c] \\
\delta(\Omega_a \Phi) &= [L_a^c, \Omega_c] \Phi - \Omega_c L_a^c \Phi, \quad \delta(\Phi \Omega^a) = -\Phi [\Omega^c, L_c^a] + \Phi L_c^a \Omega^c \quad .
\end{aligned} \tag{14}$$

Notice that these transformation rules coincide with the usual Lorentz transformations for the fields appearing in the large N continuum limit. The reader can easily verify that the inner product defined in Eq. (2) is invariant under matrix Lorentz transformations.

Likewise, consider closure under matrix Yang-Mills transformations. These will be defined so as to coincide with the usual nonabelian gauge transformation rules for the matter and gauge fields obtained in the large N limit. Consider a d_G -plet of infinitesimal real matrices, α^j , each of which takes diagonal $N \times N$ form. The matrix Yang-Mills transformations will be defined as follows:

$$\begin{aligned}
\delta\chi &= i\tau^j \alpha^j \chi, \quad \delta\bar{\chi} = -i\tau^j \bar{\chi} \alpha^j, \quad \delta(g A_g^j \tau^j) = [\Omega_a, \tau^j \alpha^j] \\
\delta(\Omega_a \Phi) &= i\tau^j \alpha^j \Omega_a \Phi, \quad \delta(\Phi \hat{\Omega}_a) = -i\tau^j \Phi \hat{\Omega}_a \alpha^j \quad .
\end{aligned} \tag{15}$$

It is easy to verify that both the classical action, \mathcal{S} , and the inner product, are invariant under matrix Yang-Mills transformations.

Closure of the group of transformations that are the finite N manifestation of large N continuum supersymmetry algebra is a nontrivial result. However, as we will see below, with the ordering prescription given earlier, the manipulations required to verify that \mathcal{S} is supersymmetry invariant are well-defined. Consider infinitesimal spinor parameters, η_1, η_2 , each of which transforms as a N -vector of the unitary group $SU(N)$. We must verify that the commutator of two matrix supersymmetry transformations with arbitrary infinitesimal spinor parameters can always be expressed as the sum of (i) an infinitesimal tangent space translation with parameter, $\xi^a = \bar{\eta}_1 \Gamma^a \eta_2$, (ii) an infinitesimal local Lorentz transformation with parameter $L_{bc} = \xi^a \omega_{abc}$, and (iii) an infinitesimal local gauge transformation with gauge parameter $\alpha^i = -g \xi^a A_a^i$ [21]. Guided by the form of the locally supersymmetric type I action [22], we arrive at the following sequence of matrix transformations induced by the infinitesimal matrix “spinor” parameter, η , an N -dimensional vector under $SU(N)$:

$$\begin{aligned}
\delta A_\mu^i &= \frac{1}{2} \bar{\eta} \Gamma_\mu \chi^i \\
\delta\chi &= -\frac{1}{2} (\Gamma^{ab} F_{ab}) \eta + \frac{1}{2} (\bar{\eta} \chi - \bar{\chi} \eta) \lambda - \frac{1}{2} (\bar{\chi} \Gamma^a \eta) \Gamma_a \lambda \\
\delta E_\mu^a &= \frac{1}{2} \bar{\eta} \Gamma^a \psi_\mu \\
\delta\psi_\mu &= D_\mu \eta + \frac{1}{2} (\bar{\eta} \psi_\mu - \bar{\psi}_\mu \eta) \lambda - \frac{1}{2} (\bar{\psi}_\mu \Gamma^a \eta) \Gamma_a \lambda + \frac{1}{g^2} (\bar{\chi} \Gamma^{abc} \chi) \Gamma_{abc} \Gamma_\mu \eta \\
\delta\Phi &= \bar{\eta} \lambda \\
\delta\lambda &= -\frac{1}{4} (\Gamma^a D_a \Phi) \eta + \left(H_{abc} - \bar{\lambda} \Gamma_{abc} \lambda + \frac{1}{g^2} \text{tr}(\bar{\chi} \Gamma_{abc} \chi) \right) \Gamma^{abc} \eta \quad .
\end{aligned} \tag{16}$$

We emphasize that there is no ambiguity in the ordering of variables in the matrix supersymmetry transformation laws as given above.

In closing, we should note that, in principle, \mathcal{S} belongs to a family of matrix actions, members of which can differ by $1/N$ corrections, thus yielding the same spacetime effective action in the infrared in accordance with the principle of Universality Classes [19, 20]. Our procedure for determining \mathcal{S} ensures that all relevant interactions in the large N continuum action that are required in

order to match correctly with the type I effective action with manifest Yang-Mills invariance, local supersymmetry, and Lorentz invariance at the scale $\alpha'^{-1/2}$, are already present in the ultraviolet theory defined by \mathcal{S} . Thus, the sole source for both nonperturbative, or quantum, corrections to the spacetime low energy effective action of the type I string are the quantum corrections from the matrix path integral.

We should emphasize that unlike the well-understood case of the hermitian one-matrix model where the angular variables decouple [13], the quantum corrections to our matrix action will be sensitive to the off-diagonal entries of the $SU(N)$ matrix variables because of the proliferation of terms in the action coupling distinct species of matrices. However, unlike the case of a generic multi-matrix model, our classical action can be motivated largely by symmetry considerations alone. This important observation rests on the existence of the finite N matrix symmetry transformations demonstrated in this paper. We save further discussion of the matrix quantum effective action for future work.

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NOTE ADDED (JULY 2005):

The basic framework given here has been fleshed out in subsequent papers, notably hep-th/0408057 [Nucl. Phys. **B719** (2005) 188]. As noted in footnote 2 of this reference, the assignment of supersymmetric partners, bosonic and fermionic, to distinct $SU(N)$ representations, namely, adjoint and vector, made in this earlier paper, was dropped by us in subsequent work. The reason is that, in such matrix models, supersymmetry does not commute with the $SU(N)$ algebra, necessitating a rather nontrivial large N limit: the correspondence to continuum target spacetime physics becomes obscure. This is also the reason we invoke $SU(N)$, distinguishing the N and \bar{N} representations, in this earlier work, while the adjoint representation of the $U(N)$ flavor group is the only species of matrix variable mentioned in subsequent papers. Recall that either unitary or hermitian matrices were acceptable in the $c \leq 1$ matrix models [15]; the distinction was found to be a moot point in the large N continuum limit of smooth Riemann surfaces, a property that was called *matrix universality*. One should note that the discretization of unorientable worldsheets does require complex matrix models. In contrast, the matrix Lagrangians given here, and in hep-th/0408057, make no reference to discretized worldsheets: they match directly to the *target spacetime* formulation of superstring theory. But perhaps use can be made of this broad class of generalized matrix models in other contexts. I would like to thank Bernard de Wit for discussions of these important distinctions. The discussion of electric-magnetic, and strong-weak coupling, dualities is a bit confusing in this paper; the reader will wonder why the full spectrum of Ramond-Ramond pform potentials is invoked at certain points in the discussion, even though at other points we only refer to the unoriented type I-I' string theories in the large N continuum limit. Notice that we have implicitly included the heterotic string theories, since they share the same perturbative string spectrum, and target space

Lagrangian, modulo duality transformations, in backgrounds with sixteen supercharges. And the generic type II pform potentials will enter into any description of type IIA or type IIB backgrounds with sixteen supercharges. Thus, the matrix Lagrangian must include all necessary pform potentials in order to incorporate these additional string backgrounds as suitable large N limits. These aspects became clearer to me in follow-up works: hep-th/0202138, 0205306, and 0210134. Finally, note that a derivation of the matrix Lagrangian appearing in this paper, by the spacetime reduction technique, is given in hep-th/0408057. A summary emphasizing electric-magnetic duality appears in hep-th/0507116.

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